

Understanding assumptions behind propensity scores versus instrumental variable estimators

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Outline

- Econometric issues in observational studies
- Focus on the Average Treatment Effect (ATE).
- The “potential outcomes” framework
- Types of biases that may arise in observational studies
- How different estimators aim to address these biases
- How these different estimators are implemented
- Empirical example based on effect of treatments on medical costs in breast cancer patients

Potential Outcomes Framework

States of the World

Two states –

Treated State: Subject having undergone a treatment

Untreated State: Subject having NOT undergone a treatment

Let the *treated* state denoted by $j = 1$ and the *untreated* state denoted by $j = 0$

Potential Outcomes Framework

Potential Outcomes

$$Y_{1i} = \beta_1 + \beta_{1X} X_i + U_{1i}$$

$$Y_{0i} = \beta_0 + \beta_{0X} X_i + U_{0i}$$

Potential Outcomes Framework

Potential Outcomes

$$Y_{1i} = \beta_1 + \beta_{1X} X_i + U_{1i}$$

$$Y_{0i} = \beta_0 + \beta_{0X} X_i + U_{0i}$$

Treatment Effect for Each Individual i :

$$Y_{1i} - Y_{0i} = \underbrace{(\beta_1 - \beta_0)}_{\text{ATE}} + \underbrace{(\beta_{1X} X_i - \beta_{0X} X_i)}_{\text{observed heterogeneity}} + \underbrace{(U_{1i} - U_{0i})}_{\text{unobserved heterogeneity}}$$

ATE
($X=0$)

*observed
heterogeneity*

*unobserved
heterogeneity*

$$\text{ATE} = \mathbf{E}(Y_1) - \mathbf{E}(Y_0) = (\beta_1 - \beta_0) + (\beta_{1X} \bar{X} - \beta_{0X} \bar{X})$$

Fundamental Problem in Evaluation

$$\underline{\text{Observed Outcome } Y = D * Y_1 + (1 - D) * Y_0}$$

So, what is the problem with observed sample means?

$$\begin{aligned} & E(Y | D = 1, X) - E(Y | D = 0, X) \\ &= E(Y_1 | D = 1, X) - E(Y_0 | D = 0, X) \\ &= E(\beta_1 + \beta_{1X}X + U_1 | D = 1) - E(\beta_0 + \beta_{0X}X + U_0 | D = 0) \\ &= \beta_1 - \beta_0 \\ &+ \beta_{1X}E(X | D = 1) - \beta_{0X}E(X | D = 0) \quad \boxed{\text{OVERT BIAS}} \\ &+ E(U_1 | D = 1) - E(U_0 | D = 0) \quad \boxed{\text{HIDDEN BIAS}} \end{aligned}$$

Econometric Methods for Observational Studies

Regression Methods

Create a model to explicitly adjust for differences in X across treatment groups. Therefore,

$$\text{Make } E(X | D = 1) = E(X | D = 0) = E(X^*)$$

Only addresses overt biases. So estimate of ATE is still as association!

Propensity Score - Theory

- Match individuals from different treatment groups so that:

$$E(X | D = 1) = E(X | D = 0)$$

- Addresses overt biases. So estimate of ATE is still as association!
- When regression model is correct
 - consistent and efficient estimate of ATE
- When regression model is incorrect
 - inconsistent estimate of ATE
- Regression model is less likely to be reliable when there are several covariates and moderate sample size.

Propensity Score - Theory

- Propensity Score methods
 - Does not rely on obtaining the correct model for the data at hand.
 - Reduces the dimensionality of the matching problem by to a single scalar quantity – the propensity score.
- Propensity Score = the probability that a subject with characteristics X chooses treatment.
- The true probability is unknown. However, the estimated probability can be used to match subjects.
- Rationale: $\Pr(D | X) \propto \Pr(X | D)$ (By conditional probability)

Propensity Score – Methods

Step 1: Estimate a logistic or a probit model

$$\log\left(\frac{p}{1-p}\right) = \alpha_0 + \alpha_1 X, \text{ where } p = E(D|X)$$

Predict the estimated propensity score for each subject in the sample = $\hat{p}(X)$

Propensity Score – Methods

Step2: Several alternative ways for matching

1. Quintiles of propensity scores

| | Avg $Y D=1$ | Avg $Y D=0$ | E_q |
|------------|----------------|----------------|--------------------------------|
| Quintile 1 | \bar{y}_{11} | \bar{y}_{01} | $\bar{y}_{11} - \bar{y}_{01}$ |
| Quintile 2 | \bar{y}_{12} | \bar{y}_{02} | $\bar{y}_{12} - \bar{y}_{02}$ |
| Quintile 3 | \bar{y}_{13} | \bar{y}_{03} | $\bar{y}_{13} - \bar{y}_{03}$ |
| Quintile 4 | \bar{y}_{14} | \bar{y}_{04} | $\bar{y}_{14} - \bar{y}_{04}$ |
| Quintile 5 | \bar{y}_{15} | \bar{y}_{05} | $\bar{y}_{15} - \bar{y}_{05}$ |
| | | TE | $\frac{1}{5} \sum_{q=1}^5 E_q$ |

2. Inverse probability weighted models

Weight treated and untreated individuals differentially:

Treated individuals = $1/\hat{p}$; Untreated individuals = $1/(1-\hat{p})$

$$TE = \sum_{i=1}^n \frac{I(D_i = 1) * Y_i}{\hat{p}_i(X)} - \frac{I(D_i = 0) * Y_i}{(1 - \hat{p}_i(X))}$$

3. Doubly robust estimators

1st Stage: Estimate propensity scores

2nd Stage: Incorporate the propensity scores as additional covariates in the regression model of outcome on D and X.

$$Y = \beta_0 + \beta_1 D + \beta_2 X + \beta_3 \hat{p}(X) + \varepsilon$$

Instrumental Variable Analysis - Theory

- Identify instrument variables that are correlated with treatment receipt but are uncorrelated with outcomes.
- Therefore, an instrument can effectively randomize subjects across treatment arms → achieve equal distribution of both X 's and U across treatment groups.
- Thus, addresses both overt and hidden biases in estimating ATE.
- Instrumental variables splits the variation in treatment variable into an exogenous part and an endogenous part

Instrumental Variable Analysis - Methods

Two-stage Methods:

Stage 1: Run propensity model same as before, but after adjusting for IVs

$$\log\left(\frac{p}{1-p}\right) = \alpha_0 + \alpha_1 X + \alpha_2 Z, \text{ where } p = E(D|X, Z)$$

Predict the estimated propensity score for each subject in the sample = $\hat{p}(X, Z)$

Compute residual for each individual: $\hat{r} = D - \hat{p}(X, Z)$

NOTE: 1st stage is a good way to check if Z significantly affects D
(F-statistics for $\alpha_2 > 10$)

Instrumental Variable Analysis - Methods

Stage 2a: (Predictor Substitution) Run outcomes regression with $\hat{\rho}$
REPLACING D :

$$Y = \beta_0 + \beta_1 \hat{\rho} + \beta_2 X + \varepsilon$$

OR

Stage 2b: (Residual Inclusion) Run outcomes regression with \hat{r} as
one of the covariates

$$Y = \beta_0 + \beta_1 D + \beta_2 X + \beta_3 \hat{r} + \varepsilon$$

Instrumental Variable Analysis - Methods

Stage 2a: (Predictor Substitution) Run outcomes regression with $\hat{\rho}$
REPLACING D :

$$Y = f(\beta_0 + \beta_1 \hat{\rho} + \beta_2 X) + \varepsilon \quad \mathbf{X} \text{ – for non-linear models}$$

OR

Stage 2b: (Residual Inclusion) Run outcomes regression with \hat{r} as
one of the covariates

$$Y = f(\beta_0 + \beta_1 D + \beta_2 X + \beta_3 \hat{r}) + \varepsilon$$

Empirical Example

AN APPLICATION IN BREAST CANCER PATIENTS

Outcomes and Preferences in Older Women Nationwide Survey (OPTIONS) project (Hadley et al., 2001)

5% random sample of all Medicare beneficiaries

Inclusions: breast cancer diagnosis or relevant surgery procedure codes for calendar years 1992 to 1994;

breast-conserving surgery with radiation (BCSRT) and mastectomy (MST) considered equivalent from the clinical point of view

Exclusions: patients with metastasis cancer

patients who were in a Medicare HMO in the month of the survey

patients who had breast-conservation surgery but did not receive radiation

Primary Outcome measure: 5-year total direct medical costs

- includes all inpatient, outpatient, and physician Part-B claims The total costs were calculated using an annual 3% discount rate.

AN APPLICATION IN BREAST CANCER PATIENTS

The final sample consisted of 2,517 patients of whom 1813 patients had a MST and the remaining had BCSRT.

Covariates:

- Treatment group,
- Age at the time of surgery, cancer stage, Charlson co-morbidity index, patient-specific Medicare payments in the year before surgery categorized into 5 groups, and race.
- % college graduates, median household income and % below poverty level by 5-digit zip-code level of the women's residence.
- County-level data on health system characteristics, such as hospital admissions, number of nursing homes and an indicator for urban area.

Instruments:

A regional dummy variable to represent Northern States,
Medicare physician fee differential between MST and BCSRT.

MODELS

- **Adjustments with EEE → GLM model with flexible link and variance parameters that are estimated from the data.**
- **Propensity Score Models:**
 - **Quintiles –based**
 - **Inverse-weighting**
 - **Doubly robust with EEE**
- **IV Models:**
 - **Predictor Substitution with EEE**
 - **Residual Inclusion with EEE**

| Model | Treatment effects Mean (sd) |
|-------------------------------|--------------------------------|
| Unadjusted | 8593 (1522) ⁺ |
| Adjusted with EEE | 10944 (1540) ⁺ |
| Propensity Scores: | |
| Quintiles based | |
| Inverse-weighting | |
| Doubly-robust with EEE | |
| IV: | |
| Predictor substitution in EEE | |
| Residual inclusion in EEE | |

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| Quintiles based | 13340 (1777) ⁺ |
| Inverse-weighting | 12604 (2398) ⁺ |
| Doubly-robust with EEE | 10676 (1563) ⁺ |
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| Doubly-robust with EEE | 10676 (1563) ⁺ |
| IV: | |
| Predictor substitution in EEE | 7286 (5877) |
| Residual inclusion in EEE | 15417 (5110) ⁺ |

Conclusions

- Regression models and propensity score matching DO NOT address hidden selection bias → so their estimates remain to be a measure of ASSOCIATION.
- IVs can address hidden selection bias → IVs work on a very effective theory that attempts to produce “true casual effects”, but the theory is not fully testable.
- Other blues of addressing hidden biases with IV:
 - Often difficult to come up with an IV (though health outcomes have better luck!)
 - Assumes a constant treatment effect conditional on observed characteristics.
 - Extensions to Local Average Treatment Effect & Local IV effects